

Markov Modeling of Fault-Tolerant Wireless Sensor Networks

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Wireless Sensor Networks (WSNs)



Ever Increasing



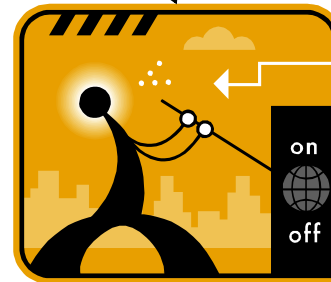
Security and Defense Systems



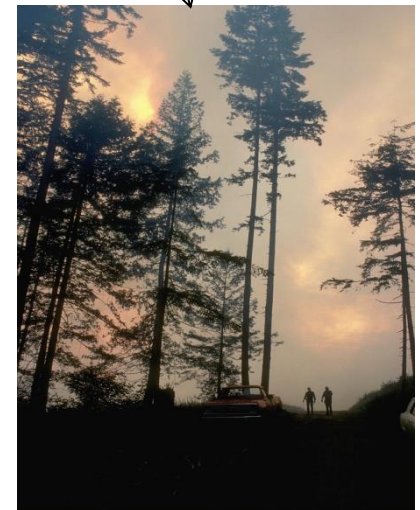
Health Care



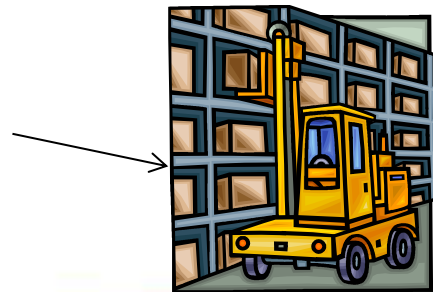
Industrial Automation



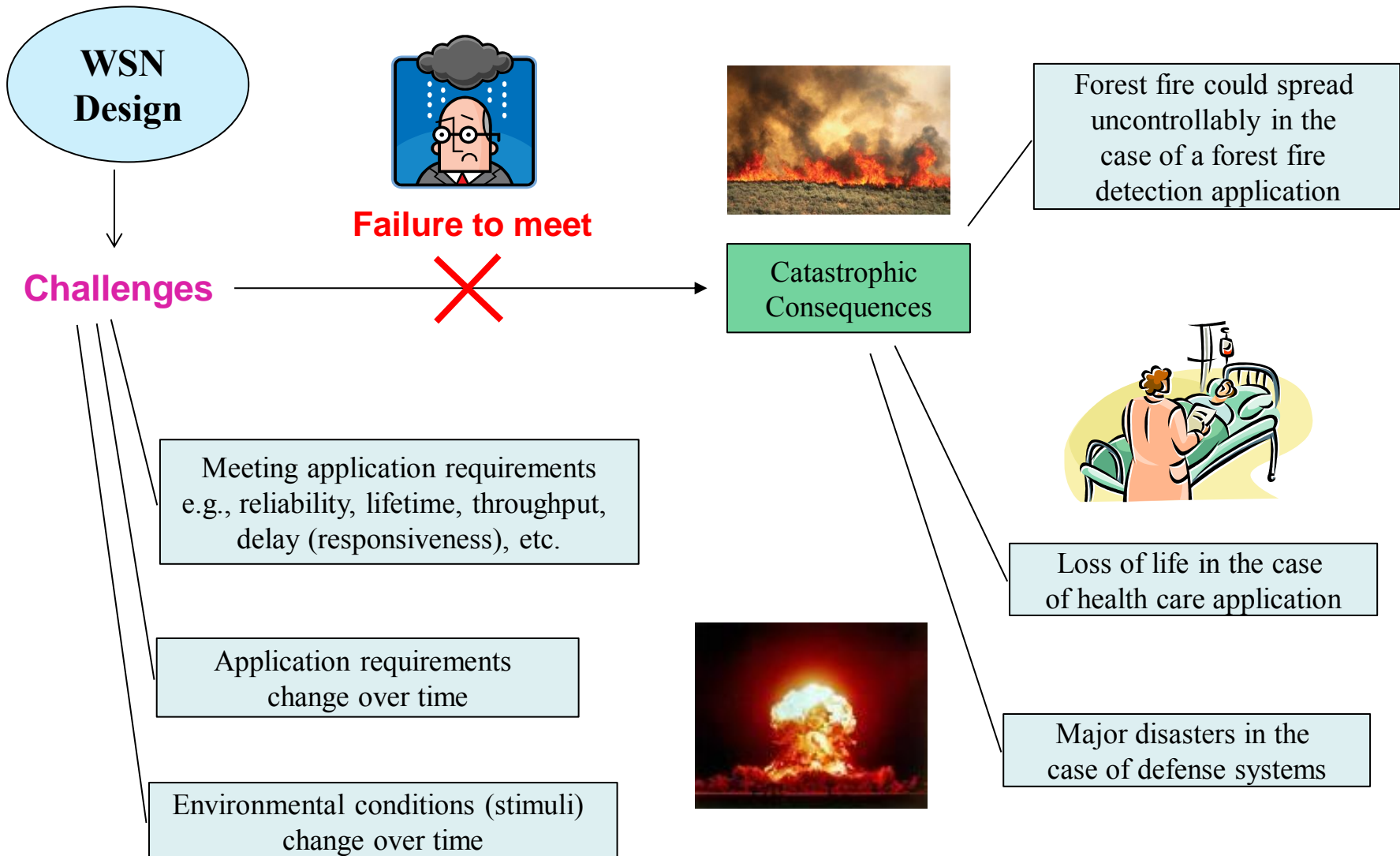
Ambient conditions Monitoring, e.g., forest fire detection



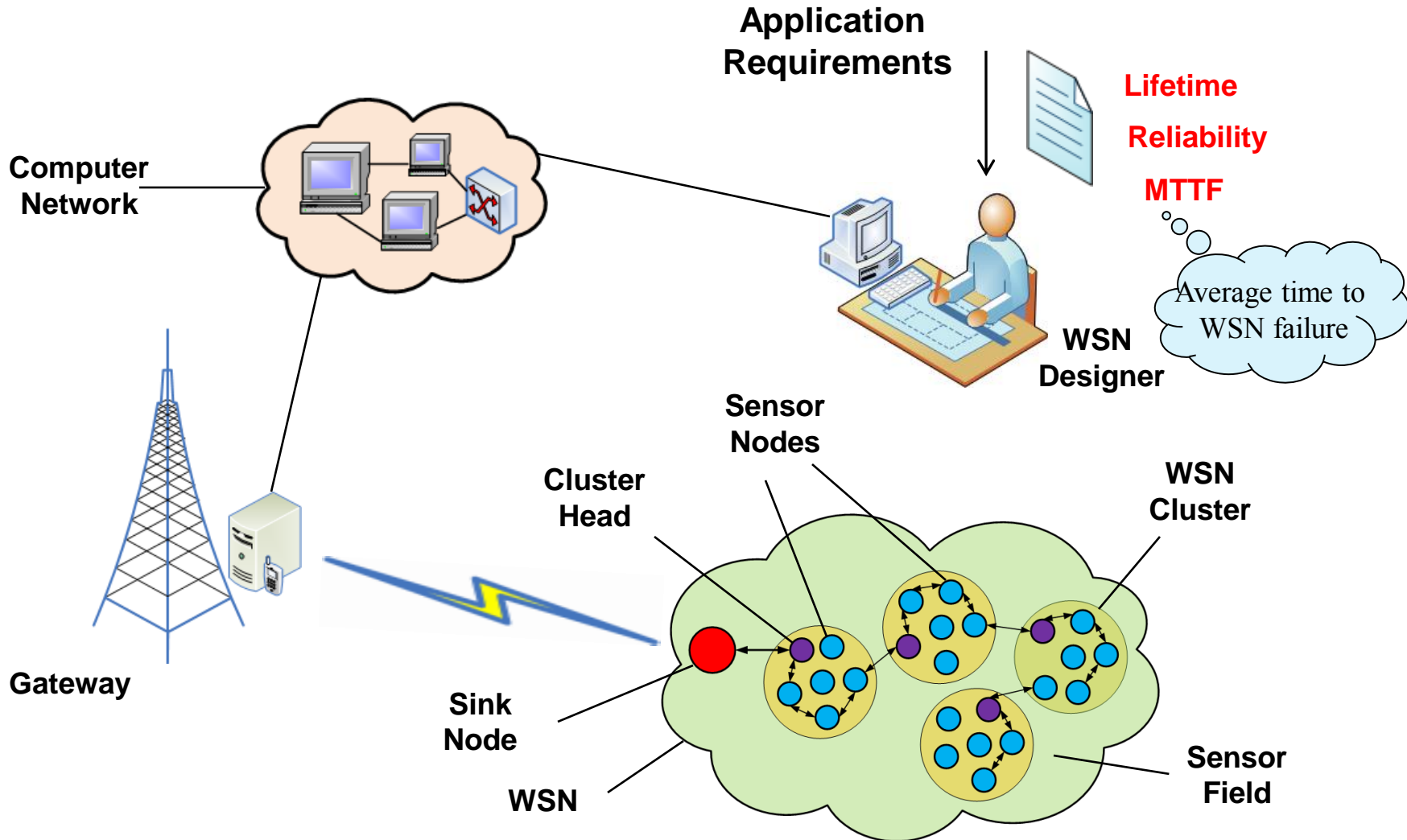
Logistics



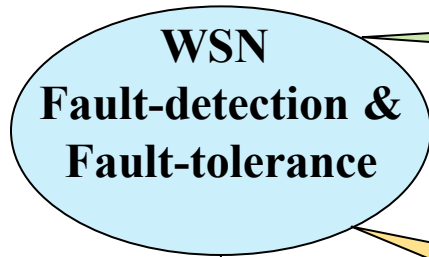
WSN Design Challenges



WSN Hierarchical View



WSN Fault-Detection and Fault-Tolerance

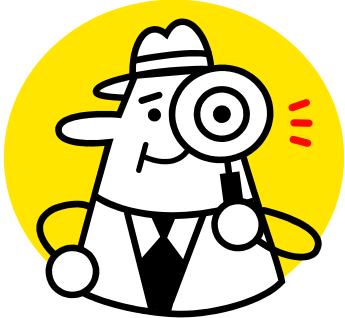


Fault-detection => Distributed fault-detection (DFD) algorithms that identify faulty sensor readings to indicate sensor faults

Fault-tolerance => Adding hardware/software redundancy to the system

DFD algorithm's accuracy => Algorithm's ability to accurately identify faults

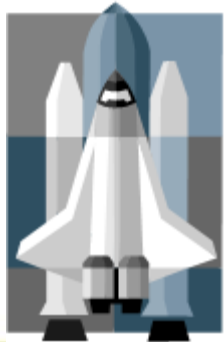
Necessity



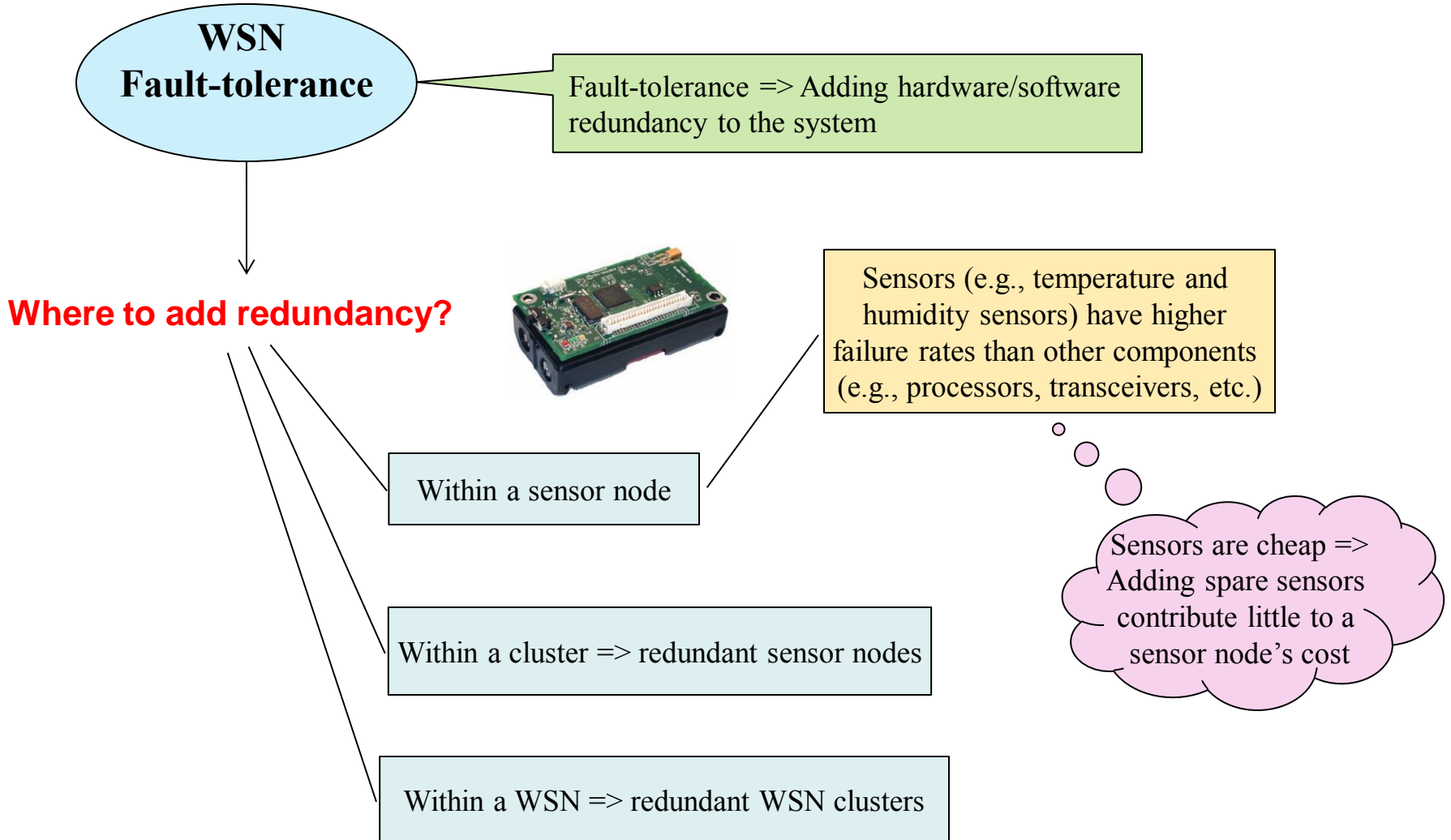
Sensor nodes deployed in unattended and hostile environments => susceptible to failures

Manual inspection of faulty sensor nodes after deployment is impractical

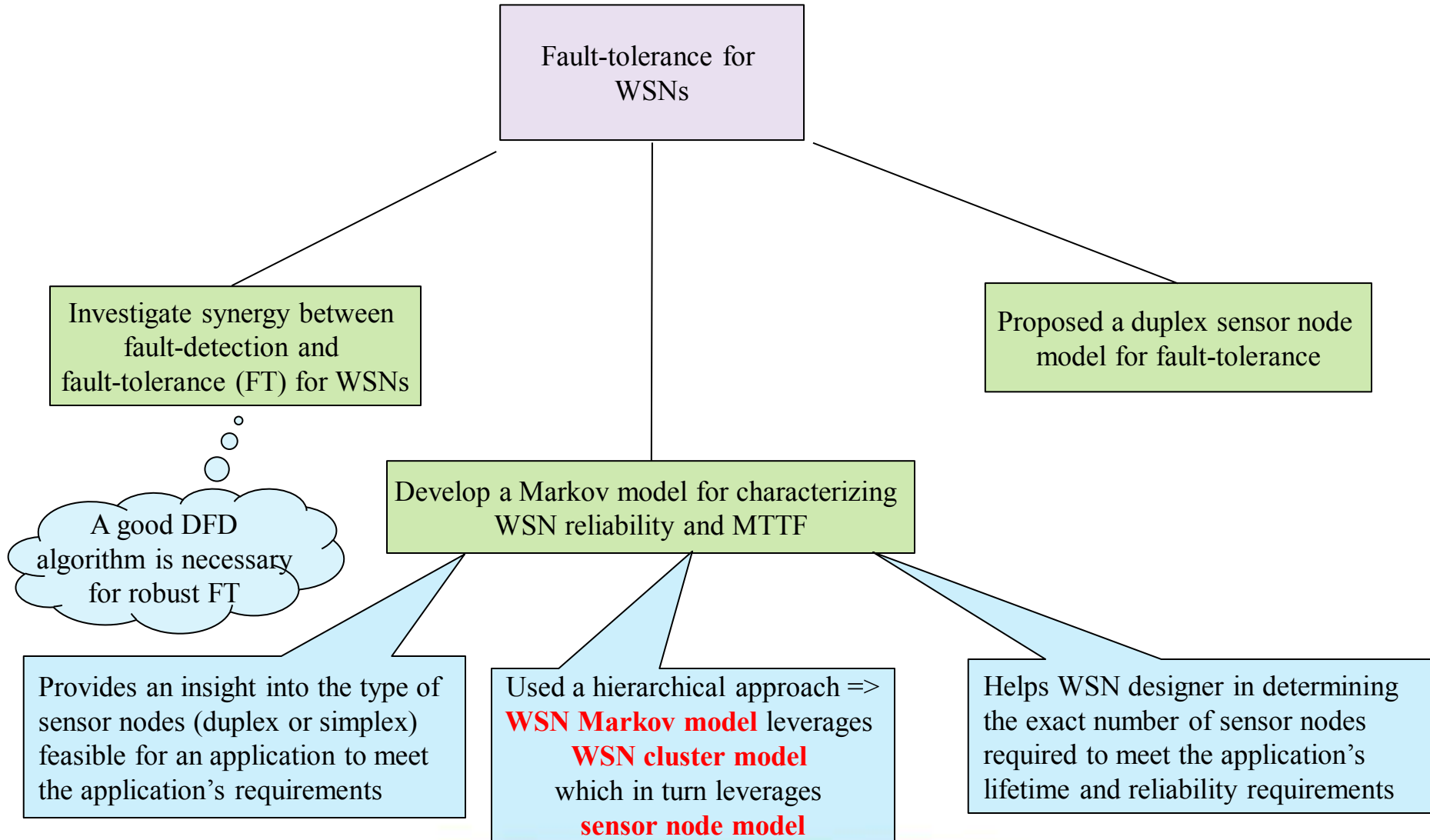
Many WSN applications are mission critical and require continuous operation



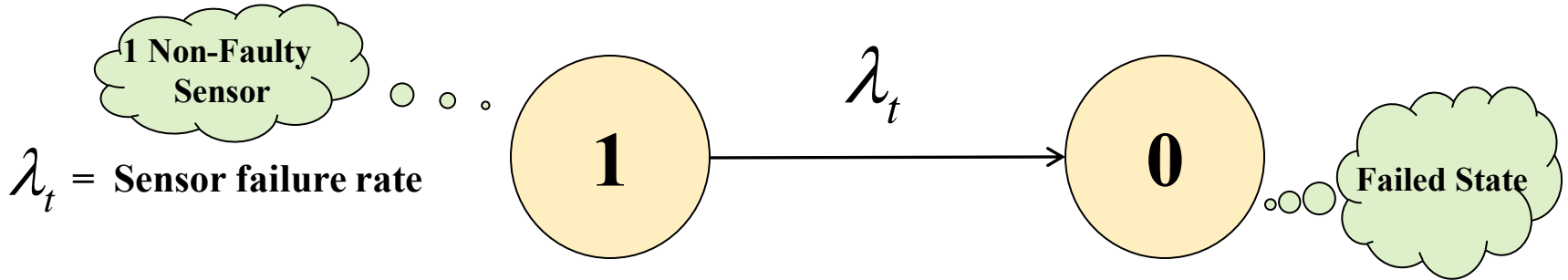
WSN Fault-Tolerance



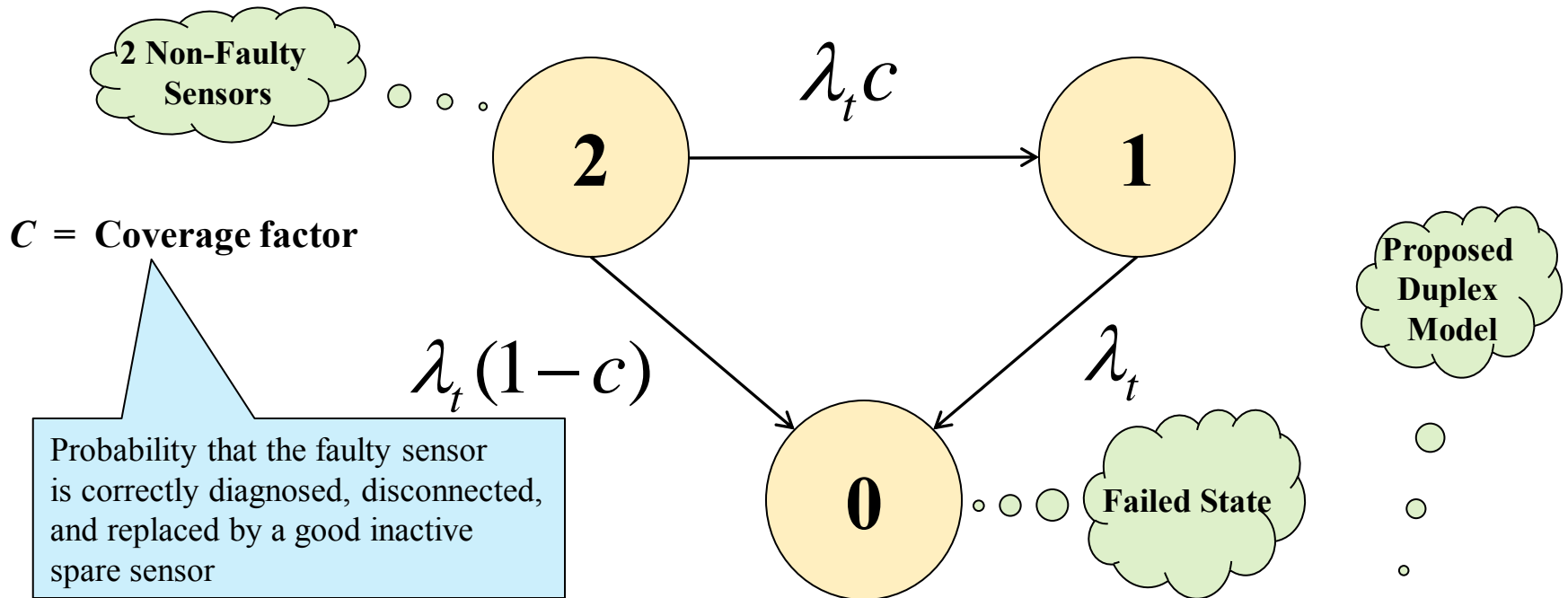
Contributions



NFT and FT Sensor Node Markov Model



Non-fault-tolerant (NFT) Sensor Node Markov Model



Fault-tolerant (FT) Sensor Node Markov Model

FT Sensor Node Markov Model – Differential Equation Solutions

- Reliability

$$\begin{aligned} R_{s_d}(t) &= 1 - P_0(t) \\ &= e^{-\lambda_t t} + c\lambda_t t e^{-\lambda_t t} \end{aligned}$$

where

- $P_i(t)$ = Probability that sensor node will be in state i at time t

- MTTF

$$\text{MTTF}_{s_d} = \int_0^{\infty} R_{s_d}(t) dt = \frac{1}{\lambda_t} + \frac{c}{\lambda_t}$$

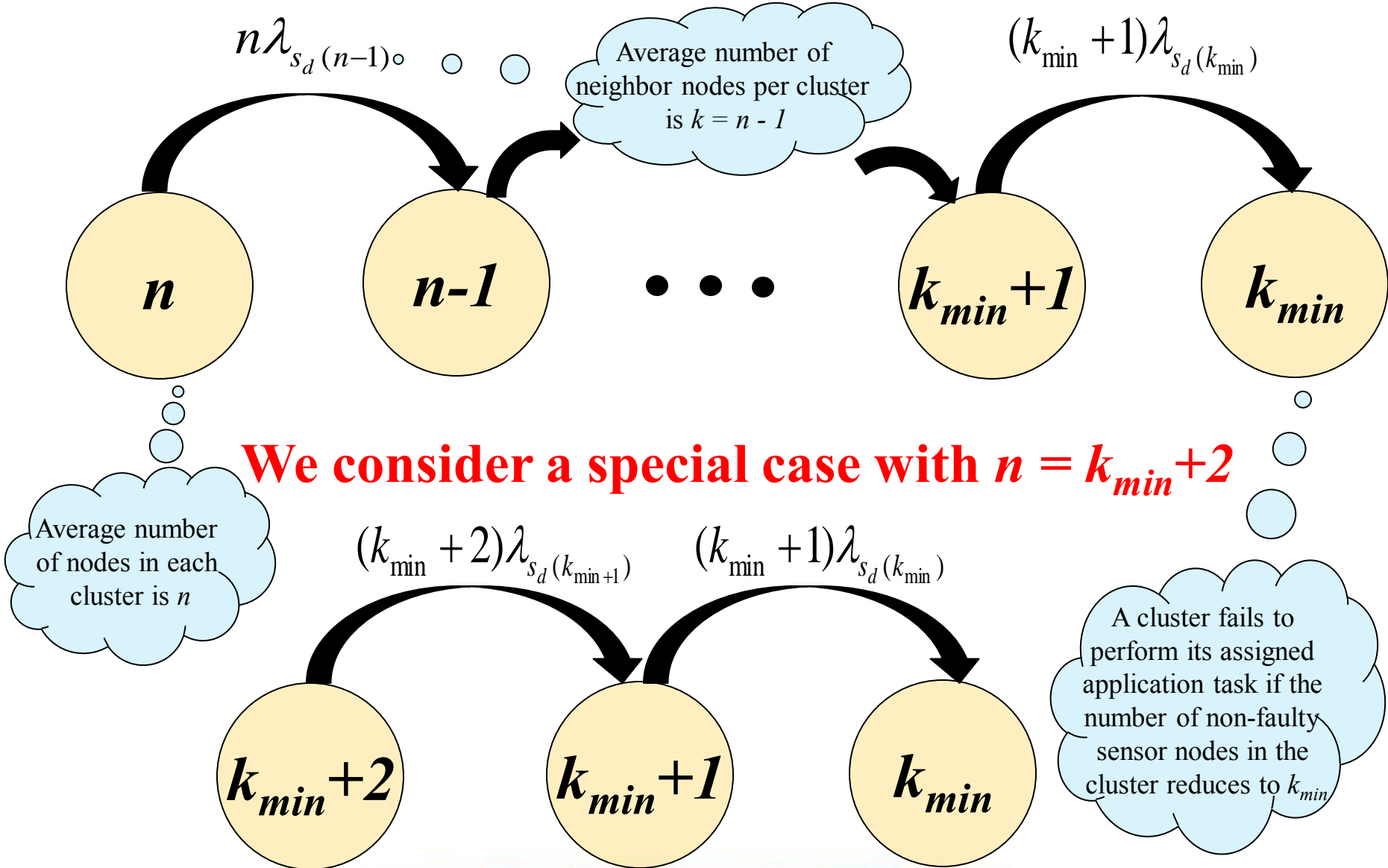
- Average Failure Rate (or Failures in Time (FIT))

$$\lambda_{s_d(k)} = \frac{1}{\text{MTTF}_{s_d(k)}}$$

where

- k denotes the average number of sensor node neighbors
- k is important as DFD algorithm's accuracy depends upon k

FT WSN Cluster Model



We consider a special case with $n = k_{min} + 2$

FT WSN Cluster Markov Model – Differential Equation Solutions ($n = k_{min} + 2$)

- Reliability

$$\begin{aligned}
 R_c(t) &= 1 - P_{k_{min}}(t) \\
 &= e^{-(k_{min}+2)\lambda_{sd}(k_{min}+1)t} + \\
 &\quad \frac{(k_{min} + 2)\lambda_{sd}(k_{min}+1)e^{-(k_{min}+2)\lambda_{sd}(k_{min}+1)t}}{(k_{min} + 1)\lambda_{sd}(k_{min}) - (k_{min} + 2)\lambda_{sd}(k_{min}+1)} + \\
 &\quad \frac{(k_{min} + 2)\lambda_{sd}(k_{min}+1)e^{-(k_{min}+1)\lambda_{sd}(k_{min})t}}{(k_{min} + 2)\lambda_{sd}(k_{min}+1) - (k_{min} + 1)\lambda_{sd}(k_{min})}
 \end{aligned}$$

- MTTF

$$\begin{aligned}
 \text{MTTF}_c &= \int_0^{\infty} R_c(t) dt = \frac{1}{(k_m + 2)\lambda_{sd}(k_m+1)} + \\
 &\quad \frac{1}{(k_m + 1)\lambda_{sd}(k_m) - (k_m + 2)\lambda_{sd}(k_m+1)} + \\
 &\quad \frac{(k_m + 2)\lambda_{sd}(k_m+1)}{(k_m + 2)(k_m + 2)\lambda_{sd}(k_m)\lambda_{sd}(k_m+1) - (k_m + 1)^2\lambda_{sd}^2(k_m)}
 \end{aligned}$$

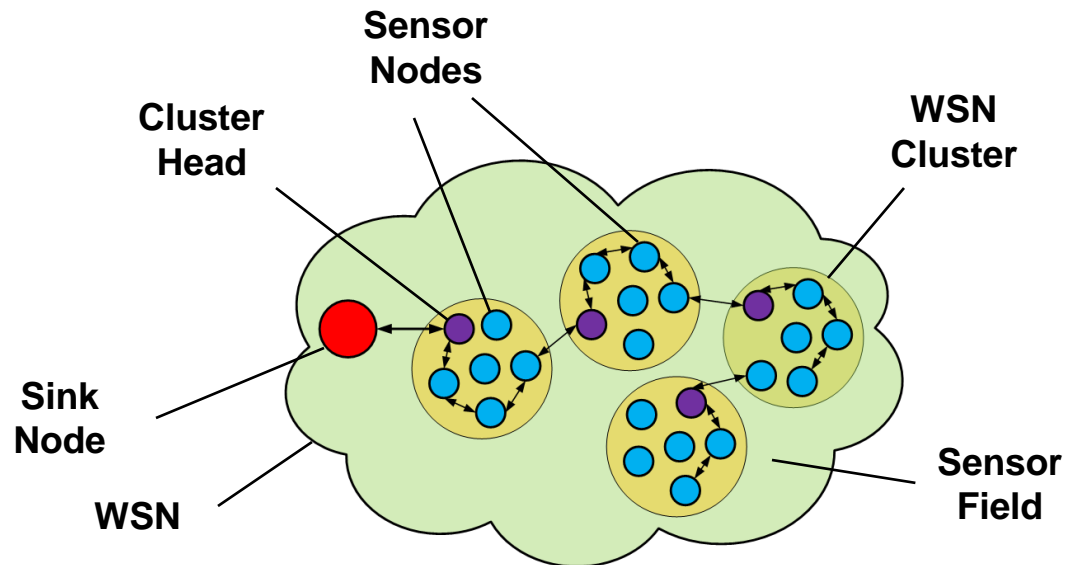
FT WSN Cluster Markov Model – Differential Equation Solutions ($n = k_{min} + 2$)

- Average Failure Rate

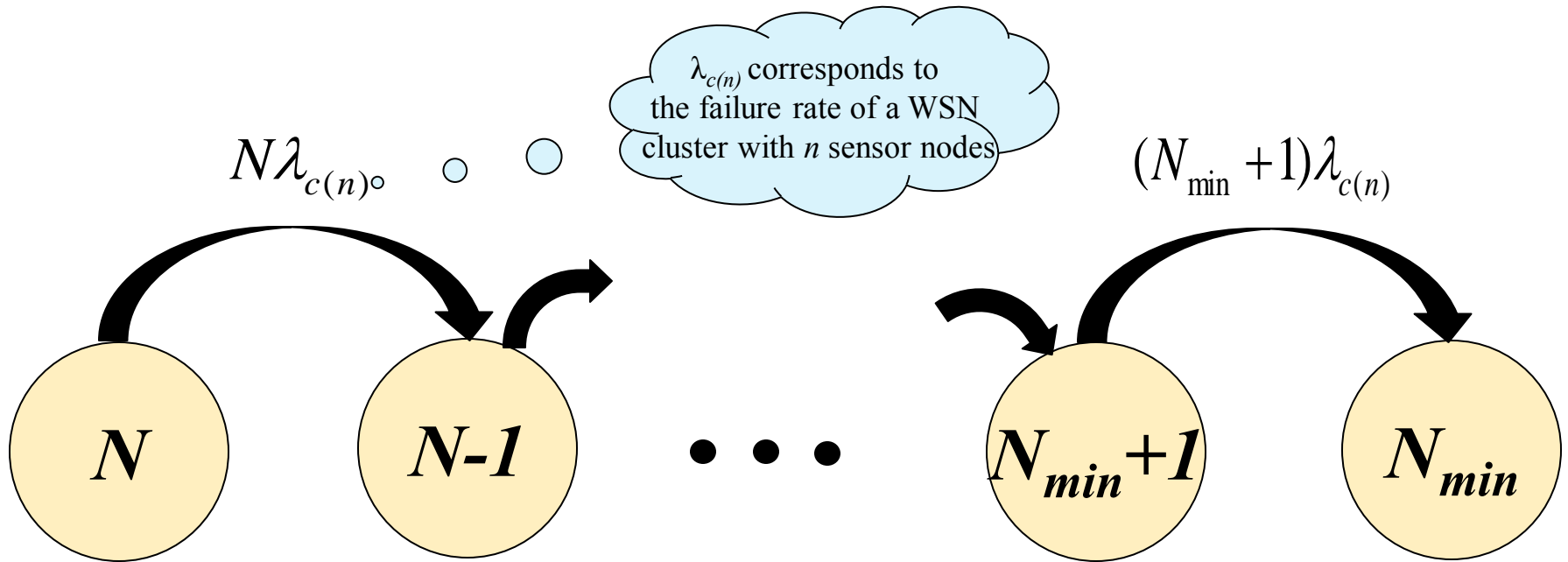
$$\lambda_{c(n)} = \frac{1}{\text{MTTF}_{c(n)}}$$

where

- $\text{MTTF}_{c(n)}$ denotes the MTTF of a WSN cluster of n sensor nodes



FT WSN Model



- A typical WSN consists of $N = n_s/n$ clusters

where

- n_s denotes the total number of sensor nodes in the WSN
- n denotes the average number of nodes in a cluster

WSN fails to perform its assigned task when the number of alive clusters reduces to N_{min}

FT WSN Markov Model – Differential Equation Solutions ($N = N_{min} + 2$)

- Reliability

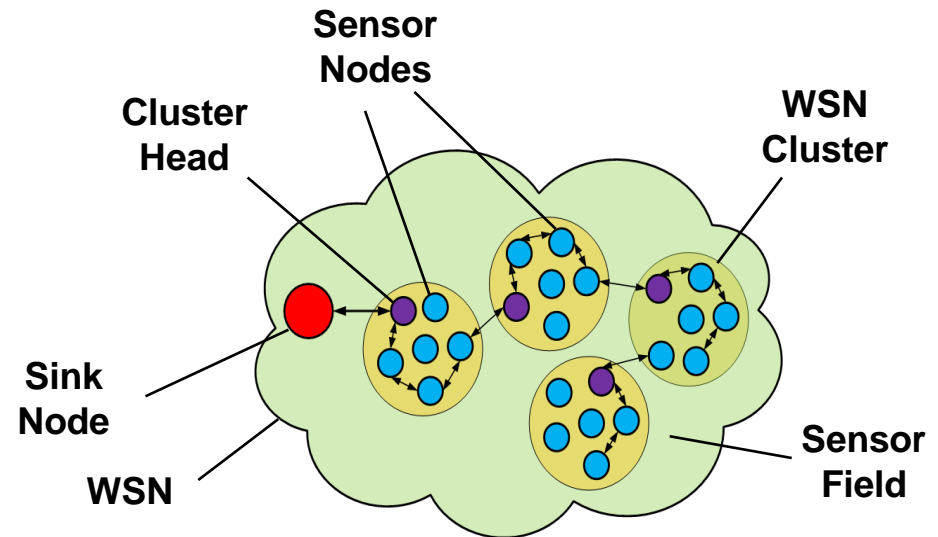
$$\begin{aligned}
 R_{wsn}(t) &= 1 - P_{N_{min}}(t) \\
 &= e^{-(N_{min}+2)\lambda_{c(n)}t} + (N_{min} + 2)\lambda_{c(n)} \times \\
 &\quad \left[e^{-(N_{min}+1)\lambda_{c(n)}t} - e^{-(N_{min}+2)\lambda_{c(n)}t} \right]
 \end{aligned}$$

- MTTF

$$\begin{aligned}
 \text{MTTF}_{wsn} &= \int_0^{\infty} R_{wsn}(t) dt \\
 &= \frac{1}{(N_{min} + 2)\lambda_{c(n)}} + \frac{N_{min} + 2}{N_{min} + 1} - 1
 \end{aligned}$$

Experimental Results

- SHARPE Software Package
 - Markov model implementations
 - NFT sensor node
 - FT sensor node
 - NFT WSN cluster
 - FT WSN cluster
 - NFT WSN
 - FT WSN



- Sensor Failure Probability
 - Exponential distribution

$$p = 1 - \exp(-\lambda_s t_s)$$

where

- p = sensor failure probability
- λ_s = sensor failure rate over period t_s
- t_s = time over which sensor failure probability/failure rate is specified
- We present results for $t_s = 100$ days

Our Markov models are valid for any other distribution as well

Results – MTTF FT & NFT Sensor Nodes

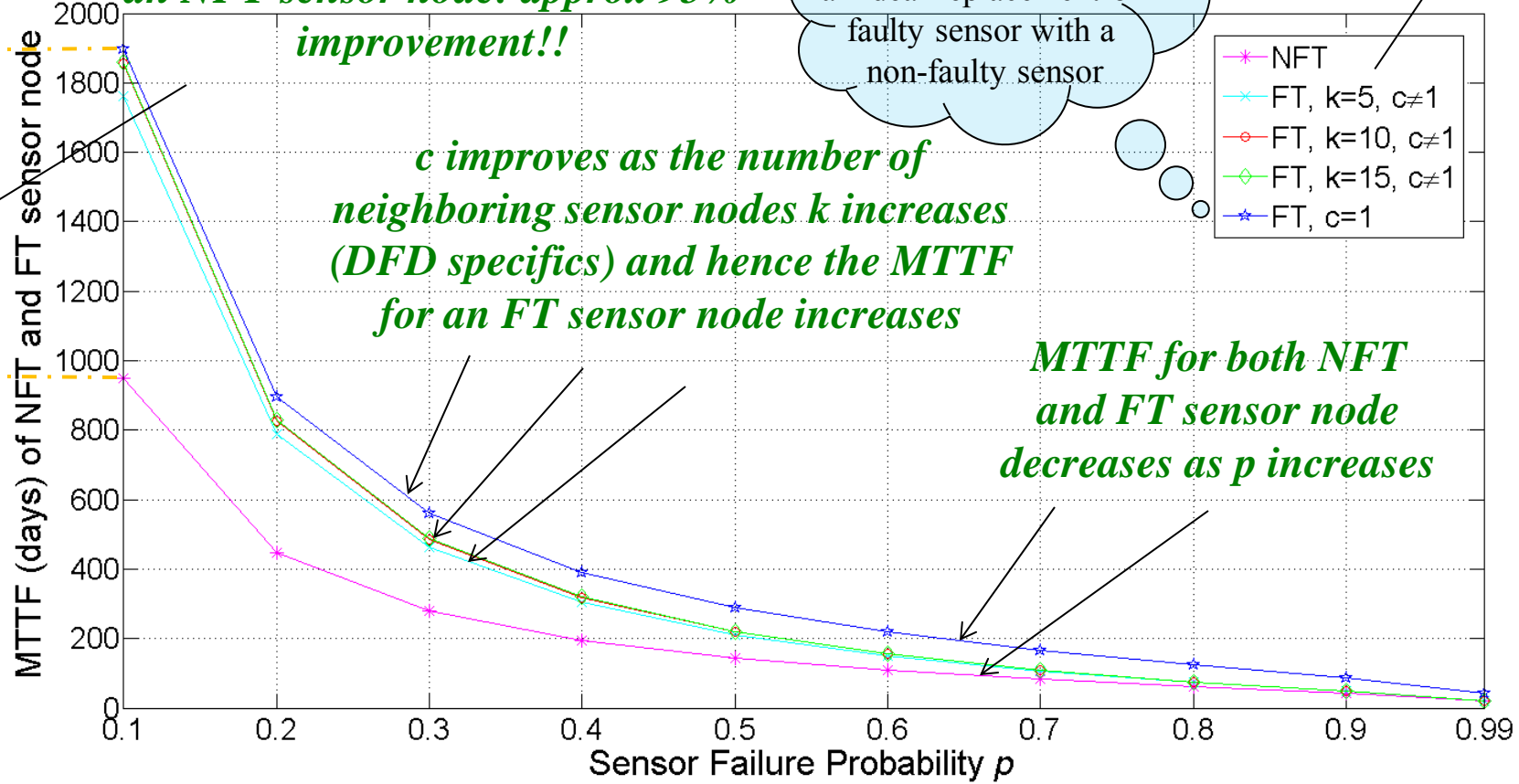
An FT sensor node has a remarkable difference in MTTF as compared to an NFT sensor node: approx 95% improvement!!

c = 1 for an ideal DFD algorithm and an ideal replacement of faulty sensor with a non-faulty sensor

c = coverage factor
k = number of neighbor sensor nodes

c improves as the number of neighboring sensor nodes k increases (DFD specifics) and hence the MTTF for an FT sensor node increases

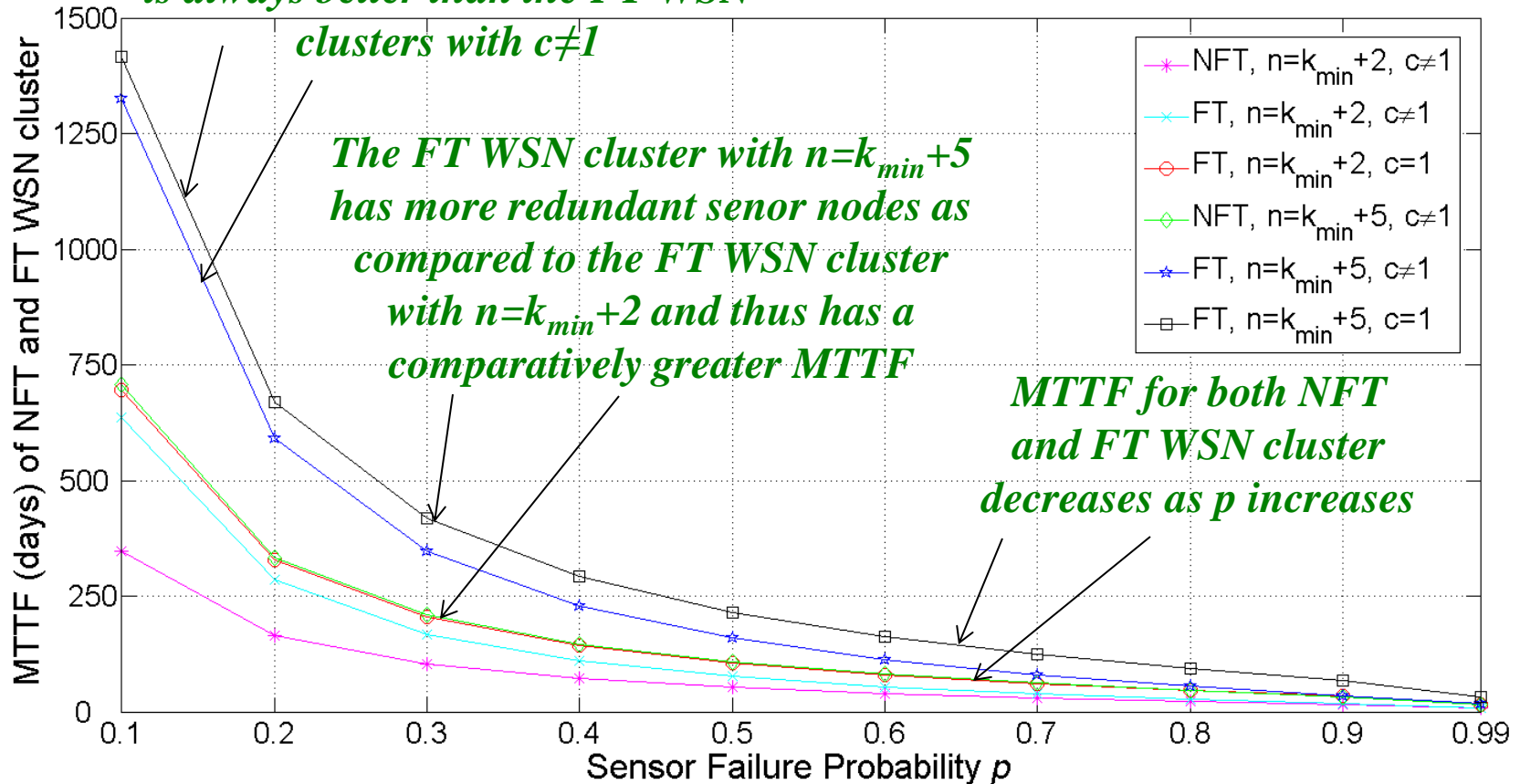
MTTF for both NFT and FT sensor node decreases as p increases



MTTF (days) for an FT and a non-FT (NFT) sensor node.

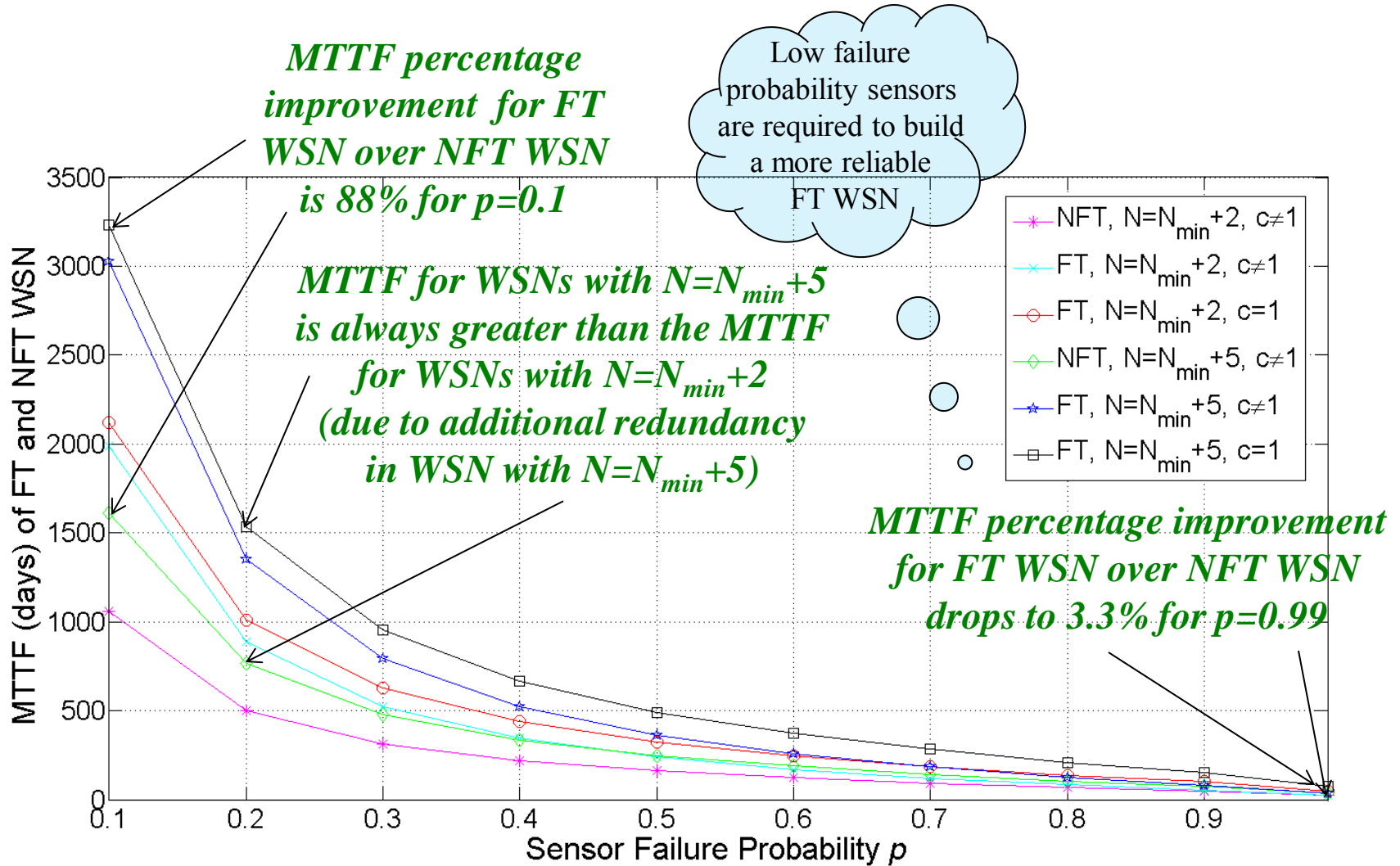
Results – MTTF FT & NFT WSN Cluster

MTTF for FT WSN clusters with $c=1$ is always better than the FT WSN clusters with $c \neq 1$



MTTF (days) for the FT and non-FT (NFT) WSN clusters with $k_{min} = 4$.

Results – MTTF FT & NFT WSN



MTTF (days) for the FT and non-FT (NFT) WSNs with $N_{min} = 0$.

Conclusions

- We proposed an FT duplex sensor node model
 - A novel concept for determining the coverage factor using sensor fault detection algorithm accuracy
- We developed hierarchical Markov models for WSNs consisting of sensor node clusters to compare the MTTF and reliability for FT and NFT WSNs
 - Aids design of WSNs with different lifetime and reliability requirements
- Fault detection algorithm's accuracy plays a critical role in FT WSNs
- FT duplex sensor node provides improvement over an NFT sensor node
 - **100%** MTTF improvement with a perfect fault detection algorithm ($c = 1$)
 - MTTF improvement from **96%** for current fault detection algorithms with low p ($p < 0.3$) - MTTF improvement reduces to 1.3% as $p \rightarrow 1$
- Percentage reliability improvement for an FT WSN with $c = 1$ over an NFT WSN with $c \neq 1$ is **350%** and over an FT WSN with $c \neq 1$ is **236%** for $p=0.9$
- **Redundancy in WSN plays an important role in improving MTTF**
 - Our models allow designers to determine the fault detection algorithm's accuracy and required redundancy to meet application requirements